Suggestions regarding Avian Mortality Extrapolation

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Introduction and Recommendations

The draft MWEC Report[1] assesses avian mortality at the Mountaineer wind farm, in Tucker County, West Virginia. During this study, birds were sometimes killed (presumably by collision with the wind turbines) and fell to the ground. Researchers visited the farm and counted carcasses at selected times. Birds on the ground are also sometimes removed by scavengers or other processes, before the researchers get to them. Experiments were also conducted to determine the searcher's efficiency (detection probability) and the scavenger removal rate. These proceeded by volitionally placing dead birds on the ground unknown to the searchers, and then seeing what happened to them.

This analysis was undertaken was as part of the author's volunteer work with the West Virginia Highlands Conservancy. The Conservancy has a seat on the Technical Review Committee for the MWEC avian monitoring activity, and thereby participated in review of the draft report. The original purpose of this analysis was to check the main extrapolation formula used in the draft report to determine the expected number of birds actually killed from the lesser number found by the researchers, and the estimates of searcher efficiency and scavenging rate. This first formula was found lacking, improved methods were devised, and these are the basis of the recommendations provided.

This author thanks Wallace P. Erickson of WEST, Inc. for his advice and participation. His work in this area has moved forward from that first referenced here, and now leads to results and methods that agree very closely to what is reported here. MWEC report authors Jessica Kerns and Paul Kerlinger are also thanked for their openness to these sorts of suggestions.

We recommend that extrapolated mortality results be reported as follows in the final MWEC report:

- Formula 1p (or the equivalent 3p) should be used and quoted as the basis for "point estimates" of total mortality. This formula assumes periodic searches, equi-spaced in time, and absolute knowledge of searcher' detection probability and scavenger's removal rate.
- That 50% and 90% confidence intervals be reported based on the Monte Carlo method developed here and the equivalent method of Erickson. These methods take into account the sample sizes used in estimation of detection probability and removal rate.

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[1] J. Kerns and P. Kerlinger, *A Study of Bird and Bat Collision Fatalities at the Mountaineer Wind Energy Center, Tucker County, West Virginia: Annual Report for 2003 DRAFT*

- That the extrapolation methodology and calculations be credited as the joint contribution of this author and Mr. Erickson.

The calculated confidence intervals reported here tend to be very broad, More than anything else, this is a consequence of the small sample size (N=29) used in estimating detection probability. We recommend that in future studies, suitably larger samples be used.

Several additional sections follow. The second section develops and discusses our first extrapolation formula, based on a Poisson (random spacing) model of the time sequence of searches. The third section replaces this with a periodic model; it is this model and formula that we now recommend. The fourth section presents the Monte Carlo method for determining confidence intervals and other statistics; its use is also recommended. The fifth section discussed the impact of some of our modeling assumptions. The sixth section presents the numerical results for several cases using parameters provided by Ms. Kerns and Mr. Erickson. These are suitable for inclusion in the final MWEC report.

Poisson Search Model

We assume here that bird deaths occur as a Poisson process, i.e., at random times, with rate λ, so that the mean time between deaths is $1/\lambda$. We assume that once a bird is killed, the event consisting of its removal by a scavenger (assuming it is not otherwise disturbed) is also Poisson, with mean time lag T_R. We assume that researcher searches for dead birds also take place as a Poisson process, with mean time between searches of T_S. These searches have detection probability p, i.e., that each bird on the ground at the time of search is found independently with this probability. We assume that all birds killed are eventually either found (and removed) by researchers or removed by scavengers. Finally, we assume that no birds are killed before time zero, and that all birds killed fall to the ground.

Let D(t) = expected number of birds killed in [0,t],
G(t) = expected number of birds on the ground at time t,
F(t) = expected number of birds found by researchers in [0,t].
R(t) = expected number of birds scavenged in [0,t].

Then

$$D(t) = \lambda t,$$
$$G(t) = D(t) - F(t) - R(t),$$
$$\text{time derivative } F'(t) = G(t)\frac{p}{T_S}, \text{ and}$$
$$\text{time derivative } R'(t) = G(t)\frac{1}{T_R}.$$

This leads to the differential equation

$$\left(\frac{p}{T_S} + \frac{1}{T_R}\right)G(t) + G'(t) = \lambda \text{ with initial condition } G(0) = 0.$$

The solution is

$$G(t) = \frac{\lambda}{a}\left(1 - e^{-at}\right), \text{ where } a = \left(\frac{p}{T_S} + \frac{1}{T_R}\right).$$

This predicts that on average dead birds will be found by researchers at an increasing rate, asymptotically approaching a steady state, determined by letting $t \to \infty$. At steady state we have

$$G(\infty) = \lambda \bigg/ \left(\frac{p}{T_S} + \frac{1}{T_R}\right) \text{and consequently} \quad F'(\infty) = G(\infty)\frac{p}{T_S} = \frac{p\lambda T_R}{pT_R + T_S}.$$

Now consider a steady-state time interval of duration T_T. Define N_K as the expected number of birds killed during this interval and N_F as the expected number of birds found by researchers during this same interval. Then we have $N_K = \lambda T_T$ and $N_F = F'(\infty)T_T$, so that

$$N_K = N_F \frac{pT_R + T_S}{pT_R}. \tag{1}$$

The draft MWEC report uses for this extrapolation a 1998 formula by Erickson et al[2]

$$m = \frac{N * I * C}{k * t * p} \tag{2}$$

In this equation, N is equal to the total number of turbines, I is the interval between searches in days, C is the total number of carcasses detected for the period of study, k is the number of turbines sampled, *t* is the mean carcass removal time in days, and p is the observer efficiency rate.

Equating their notations to ours, we have $N_F = C$, $p = p$, $T_R = t$, and $T_S = I$. They also scale up the estimate by the ratio (N/k) of total turbines to turbines sampled (in the MWEC study these two numbers were the same). Re-writing our formula in their notation, we get, instead of (2),

$$m = \frac{N * (t * p + I) * C}{k * t * p}. \tag{3}$$

[2] Erickson, W.P., M. Dale Strickland, G.D. Johnson, and J.W. Kern. Examples of statistical methods to assess risks of impacts to birds from wind plants. Proceedings of the Avian-Wind Power Planning Meeteing III. National Wind Coordinating Committee Meeting, May 1998, San Diego, CA, Washington, DC.

When applied to the draft MWEC inputs (N=k=44C=35), t = 6.7 days, p = .276, I = 7.0 days, this leads to an extrapolation of m = 167.5, vs. the MWEC report's value of 132.5, obtained with formula (2), a discrepancy of about 23%.

Our result (formula (1) or (3)) has been confirmed by Monte Carlo simulation. Erickson et al provided neither a derivation nor a reference for formula (2).

It is easily seen that, under the assumptions stated, formulas (1) and (3) provide unbiased estimates of number killed.

Periodic Search Model

We next tried a simulation where the search event repetition was periodic instead of Poisson. This did lead to a decrease in the estimate for m, to about 153.

Since in reality there was an imperfect effort to conduct the searches periodically, they were neither Poisson nor periodic, but somewhere in between. Therefore the correct value for m is thought to be in the range 155-165. Thus the number 132.5 in the report appears to be roughly 15-20% low.

The case of periodic, instantaneous search has been worked out analytically also and yields an estimate of 152.34, closely matching the Monte Carlo estimate of 153. The formula corresponding to (1) is

$$N_K = N_F\left(\frac{T_s}{pT_R}\right)\left(\frac{e^{T_s/T_R}-1+p}{e^{T_s/T_R}-1}\right) \tag{1p}$$

and that corresponding to (3) is

$$m = \left(\frac{N*I*C}{k*t*p}\right)\left(\frac{e^{I/t}-1+p}{e^{I/t}-1}\right) \tag{3p}$$

These last equations are derived as follows. We assume that the initial transient has died out by time zero and that searches occur thereafter instantaneously at times 0, T_S, $2T_S$, $3T_S$, … . Then G(t) will decline discontinuously by amount pG(t) at these times and in the intervals $(0,T_s]$, $(T_S,2T_S]$, $(2T_S,3T_S]$, etc. will satisfy the differential equation obtained from that above by setting p = 0, i.e.,

$$\left(\frac{1}{T_R}\right)G(t)+G'(t)=\lambda \ .$$

Solving this equation and requiring $G(0) = G(T_S)$ leads to an expression for the change in G(t) at the discontinuities. This is

$$\Delta G = \left(p\lambda T_R\right)\left(\frac{e^{T_s/T_R}-1}{e^{T_s/T_R}-1+p}\right) .$$

Since ΔG is also the expected number of dead birds found at each of times times 0, T_S, $2T_S$, $3T_S$, … this also gives us the long term average rate at which dead birds are found, and finally leads to equation (1p).

It is again easily seen that, under the assumptions stated, formulas (1p) and (3p) provide unbiased estimates of number killed.

Development of Confidence Intervals

A weakness in formulas (1) and (1p) is that they assume perfect knowledge of the quantities T_R and p. Actually, these are experimentally determined, sometimes with rather small samples. This introduces both bias and uncertainty in the extrapolation of number killed via (1) or (1p). For this reason, for the determination of we developed a Monte Carlo method that takes account of the sample sizes for T_R and p.

The Monte Carlo algorithm uses the following parameters:

N_p= sample size in estimation of detection probability p

N_d= number of successful detections in determination of detection probability

$\hat{p} = N_d / N_p$

N_R= sample size in estimation of expected time T_R for scavenging to occur

$\hat{T}_R$ = estimate used for T_R

$\hat{N}_F$ = number of dead birds found

The algorithm computes a large number of estimates of number killed, using equation (1p), each time using values for p, T_R, and N_F drawn from distributions for these quantities as follows:

- Draw detection probability p as a binomial random variable with parameters N_p and $\hat{p}$
- Draw scavenging time T_R as a normal random variable with parameters $\mu = \hat{T}_R$ and $\sigma = \hat{T}_R / \sqrt{N_R}$. The rationale here is that T_R is an average of N_R instances of an exponential random variable with mean and standard deviation $\hat{T}_R$.
- Draw N_F as a Poisson random variable with mean $\hat{N}_F$,
- Apply formula (1p) to get an estimate of number killed N_K.

The resultant table of values is then treated as a sample from the distribution for N_K and used to get point estimates and confidence intervals.

Impact of Assumptions

Our "unbiased estimators" assume knowledge of three parameters. These are the mean time between searches T_S, the mean time to scavenge T_R and the searcher's detection probability p. In actuality, T_S is known, but T_R and p are estimated through a common experiment. In that experiment, dead birds are volitionally placed in the field and left there until removed by scavengers, giving rise to the estimate of T_R. Meanwhile, the searcher's are sent into the field, and their detection probability p is estimated through their success in finding these same dead birds. The constants of proportionality between numbers found and estimated numbers killed in formulas (1) and (1p) are both non-linear functions of T_R and p; this is a probable source of bias in estimating number killed. This is one of the reasons we put together a Monte Carlo method for getting the confidence intervals.

We've assumed that the search schedules are either Poisson (formula (1)) or periodic (for formula (1p)). A periodic schedule is probably optimal for minimizing variance in the number killed. However, this can not be generally achieved; irregularities in the schedule actually executed are sometimes unavoidable. However, such irregularities can and should be treated by Monte Carlo if they are significant. It is also possible that an analytic formula could be obtained which would treat arbitrary search schedules.

We've also assumed a steady state in deriving our results—as if the processes of bird death, scavenging, and searching had gone on indefinitely and uniformly. We think this is a fairly reasonable assumption, since it should not take long for such a steady state to be reached.

Numerical Results

Calculations were carried out for many of the same cases considered by Erickson[3] to develop confidence intervals for the MWEC study, using his more recent methodology developed for other avian mortality studies. These are tabulated below.
Our formula (1p) was then used to provide the point estimate of total mortality. Our Monte Carlo algorithm described above was used to calculate the 50% and 90% confidence intervals, and the standard error. 10,000 replications were used in all cases, except that 100,000 replications were used in the calculation for fall bat mortality.

The results are in very close agreement with those found by Erickson using his current methods.

	Average Interval between searches (days)	No. found	Point estimate	Standard error	50% Confidence Limits		90% Confidence Limits	
All Birds Excluding the May Event								
Spring	9	11	59	37	44	81	27	130
Summer	27	1	17	20	0	25	0	54
Fall	7	24	104	55	84	138	60	215
All Birds excluding the May Event and EUST, HOSP, RODO								
Spring	9	9	48	32	35	67	21	111
Fall	7	24	104	59	84	139	60	219
Nocturnal Migrating Songbirds Excluding the Event								
Spring	9	5	27	19	18	38	8	64
Fall	7	17	74	41	58	99	40	156
Bats								
Spring	9	17	91	54	71	123	48	194
Fall	7	458	1993	971	1683	2576	1326	3918

**All calculations assume an average 6.7 days for removal by scavengers, searcher detection probability p=.276, a sample size of 29 for estimation of p, and 30 samples for removal rate estimation

[3] W.P. Erickson, Personal Communication, March, 2004